New High-Frequency Circuit Model for Coupled Lossless and Lossy Waveguide Structures

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Abstract —A coupled transmission line model is proposed describing the two fundamental modes of any two-conductor (above a ground plane or shielded) dispersive or nondispersive lossless waveguide system. The model is based on a power–current formulation of the impedances but does not need an *a priori* supposition about the power distribution over each transmission line. In the second part of the paper the analysis is extended to lossy structures and to the multiconductor situation. Impedance calculations for a typical coupled microstrip configuration are used to illustrate the approach.

I. Introduction

The design and the accurate modeling of interconnections have gained increasing importance due to their presence in high-speed electronics and MMIC's. For the calculation of the dispersion characteristics of microstrip and stripline multiconductor configurations a very large number of papers have been published. We refer the reader to [1]–[8]. A comprehensive review of papers up to October 1985 can be found in [8]. Another type of printed board is the wire board [9]. The quasi-TEM analysis for this structure and the full-wave calculation of the dispersion characteristics have been presented in [10] and [11].

In order to include the behavior of these interconnections in circuit simulators which are able to handle these different interconnection types together with (nonlinear) loads and drivers, it is imperative to come to a suitable (coupled) transmission line representation of the interconnections. This problem for a single dispersive waveguide structure is thoroughly discussed in [12] and [13]. The coupled waveguide case, however, is treated by a limited number of authors. In order to obtain a complete equivalent transmission line representation not only the dispersion characteristics must be calculated, but also the characteristic impedances. This important issue has been extensively discussed in the literature, e.g., in [1], [2], [5], [6], and [14]. In [4] Jansen proposed an equivalent coupled transmission line model for a coupled microstrip configuration based on an *a priori* given distribution of the total propagating power over the strips.

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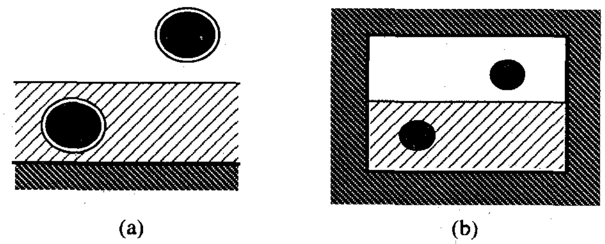

Fig. 1. Typical cross section of an (a) open and (b) closed coupled hybrid waveguide configuration.

In the first part of this paper a new coupled transmission line model is proposed describing the two fundamental modes of any two-conductor dispersive or nondispersive lossless waveguide system. The two conductors are placed above a ground plane or are shielded (see Fig. 1). Our model does not require any assumptions regarding the power distribution. Although it is also based on a power–current formulation for the impedances, it is shown that our approach only coincides with the results obtained in [4] in the quasi-TEM limit. As our model is derived from first principles only, it remains valid for any frequency.

In the second part of the paper, our model is extended to lossy structures and to the multiconductor situation. Finally, to exemplify our approach, we analyzed a typical coupled microstrip configuration. The impedance calculations were implemented in two different ways: a first implementation uses the formulas derived in this paper; a second implementation uses the formulas proposed in [4].

II. General Representation of the Fields in a Hybrid Waveguide

Fig. 1 shows typical cross sections of hybrid waveguide configurations. The geometries under consideration are independent of the x direction. The perfectly conducting lines 1 and 2 are embedded in a multilayered lossless dielectric. In the sequel our interest will be focused on the two fundamental eigenmodes propagating along the x axis of such structures. Starting from Maxwell's equations it can be shown that the total fields due to these fundamen-

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tal modes can be written as [12]

$$\begin{aligned} E(x,y,z) &= V_{v,1}(x)E_{t,1}(y,z) + R_0 I_{v,1}(x)E_{l,1}(y,z) \\ &\quad + V_{v,2}(x)E_{t,2}(y,z) + R_0 I_{v,2}(x)E_{l,2}(y,z) \\ H(x,y,z) &= I_{v,1}(x)H_{t,1}(y,z) + V_{v,1}(x)/R_0 H_{l,1}(y,z) \\ &\quad + I_{v,2}(x)H_{t,2}(y,z) \\ &\quad + V_{v,2}(x)/R_0 H_{l,2}(y,z). \end{aligned} \tag{1}$$

The electric and magnetic fields in (1) are the modal fields which depend only upon the transversal coordinates y and z. The index t refers to the transversal y and z components while the index l refers to the longitudinal x component. The voltages $V_{v,j}$ and currents $I_{v,j}$ $(j=1,2)$ are the field voltages and field currents and R_0 is the free-space impedance. The field voltages and currents satisfy [12]

$$\frac{dV_{v,j}(x)}{dx} = -j\beta_j Z_j I_{v,j}(x), \qquad j=1,2 \tag{2}$$

and

$$\frac{dI_{v,j}(x)}{dx} = -j\beta_j / Z_j V_{v,j}(x), \qquad j=1,2. \tag{3}$$

The propagation constants of each mode are denoted by β_j $(j=1,2)$. The general solution of (2) and (3) is

$$\begin{aligned} V_{v,j} &= C_j \exp(-j\beta_j x) + D_j \exp(j\beta_j x) \\ I_{v,j} &= (1/Z_j)\left[C_j \exp(-j\beta_j x) - D_j \exp(j\beta_j x)\right] \end{aligned} \tag{4}$$

where C_j and D_j are arbitrary constants. As we first restrict the analysis to lossless structures, the transverse modal fields and the impedances Z_j $(j=1,2)$ can be chosen to be real, while the longitudinal fields are purely imaginary. For this lossless situation it can be shown [15], starting from the Lorentz reciprocity theorem, that the eigenmodes 1 and 2 are power orthogonal, i.e.,

$$\iint_S (E_{t,1} \times H_{t,2}^*) \cdot dS = \iint_S (E_{t,2} \times H_{t,1}^*) \cdot dS = 0. \tag{5}$$

The total power propagated by the structure is the sum of the power propagated by each individual mode.

III. Circuit Representation of the Eigenmodes

It is our purpose to find a suitable and consistent circuit representation of the eigenmodes described in the above section in terms of a coupled transmission line (CTL) representation. An elaborate analysis for a single conductor by Brews has shown that a circuit representation in the form of a single transmission line is possible but that, generally speaking, it is impossible to give a circuit interpretation to both the current and the voltage on this transmission line equivalent. Moreover, depending upon whether a special circuit meaning is assigned to the voltage or to the current, a different value of the characteristic impedance is found. The reader is referred to [14] for further clarification and for an example showing the different characteristic impedance values obtained depending on whether a circuit meaning is assigned to the total longitudinal current or to the center voltage in a simple microstrip configuration. It is only in the low-frequency or quasi-static limit that a circuit meaning can be assigned to both the voltage and the current, as confirmed by the fact that the different impedance curves coincide in this limit.

In the sequel the following choices are made. As circuit currents in our CTL representation we choose the total longitudinal currents flowing along conductors 1 and 2. It has been extensively argued in the literature [4], [5] that this choice must be preferred if the coupled transmission line representation has to be used in conjunction with TEM models of loads and drivers. A second choice is that the total complex power propagated by the CTL model must be the same as in the actual waveguide configuration.

From the expression for the magnetic field in (1) the total longitudinal currents along conductors 1 and 2 are

$$\begin{aligned} I_1(x) &= \left[\int_1 H_{t,1} \cdot dl\right] I_{v,1}(x) + \left[\int_1 H_{t,2} \cdot dl\right] I_{v,2}(x) \\ I_2(x) &= \left[\int_2 H_{t,1} \cdot dl\right] I_{v,1}(x) + \left[\int_2 H_{t,2} \cdot dl\right] I_{v,2}(x). \end{aligned} \tag{6}$$

The integrations in (6) extend over the boundaries of the conductors as shown in Fig. 1. I_1 and I_2 represent the circuit currents in the CTL representation. Introducing a shorthand notation for the integrals, (6) can be rewritten as

$$\begin{aligned} I_1(x) &= i_{11} I_{v,1}(x) + i_{12} I_{v,2}(x) \\ I_2(x) &= i_{21} I_{v,1}(x) + i_{22} I_{v,2}(x). \end{aligned} \tag{7}$$

The circuit voltages V_1 and V_2 are still unknown, but as the total fields are a superposition of the fields coming from each mode, these voltages should be expressed as a linear combination of the field voltages:

$$\begin{aligned} V_1(x) &= v_{11} V_{v,1}(x) + v_{12} V_{v,2}(x) \\ V_2(x) &= v_{21} V_{v,1}(x) + v_{22} V_{v,2}(x). \end{aligned} \tag{8}$$

The coefficients v_{ij} $(i,j=1,2)$ are still unknown.

We now introduce the second choice on which our circuit model is based, namely the equivalence of the total propagating power. Taking into account the power orthogonality of the modes (eq. (5)), the total power propagated by the coupled waveguide structure is

$$P = 1/2\left[V_{v,1} I_{v,1}^* \iint_S (E_{t,1} \times H_{t,1}^*) \cdot dS + V_{v,2} I_{v,2}^* \cdot \iint_S (E_{t,2} \times H_{t,2}^*) \cdot dS\right]. \tag{9}$$

The integration in (9) extends over the total cross section S of the waveguide structure. Introducing the following notation:

$$p_j = \iint_S (E_{t,j} \times H_{t,j}^*) \cdot dS, \qquad j=1,2 \tag{10}$$

(9) can be rewritten as

$$P = 1/2(p_1 V_{v,1} I_{v,1}^* + p_2 V_{v,2} I_{v,2}^*). \tag{11}$$

Starting from (7) and (8), the total power propagated by the CTL model is found to be

$$P = 1/2[V_{v,1}I^*_{v,1}(v_{11}i_{11}+v_{21}i_{21}) + V_{v,1}I^*_{v,2}(v_{11}i_{12}+v_{21}i_{22}) \cdot V_{v,2}I^*_{v,1}(v_{12}i_{11}+v_{22}i_{21}) + V_{v,2}I^*_{v,2}(v_{12}i_{12}+v_{22}i_{22})] \quad (12)$$

where we have explicitly taken into account the real nature of i_{ij}. The equivalence of (11) and (12) yields the following equations:

$$\begin{aligned} v_{11}i_{11}+v_{21}i_{21} &= p_1 \\ v_{12}i_{12}+v_{22}i_{22} &= p_2 \\ v_{11}i_{12}+v_{21}i_{22} &= 0 \\ v_{12}i_{11}+v_{22}i_{21} &= 0. \end{aligned} \quad (13)$$

In order to obtain (13) we have taken into account the fact that the equality between (11) and (12) must also be satisfied for any linear combination of the two modes. From (13) it follows that v_{ij} can be found in terms of i_{ij}:

$$\begin{aligned} v_{11} &= p_1 i_{22}/\Delta i \\ v_{21} &= -p_1 i_{12}/\Delta i \\ v_{12} &= -p_2 i_{21}/\Delta i \\ v_{22} &= p_2 i_{11}/\Delta i \end{aligned} \quad (14)$$

where

$$\Delta i = i_{11}i_{22} - i_{12}i_{21}. \quad (15)$$

To get a first CTL model it now suffices to introduce (4) into (7) and (8), taking into account (14). It is found that

$$\begin{vmatrix} V_1(x) \\ V_2(x) \end{vmatrix} = \begin{vmatrix} p_1 i_{22}/\Delta i \\ -p_1 i_{12}/\Delta i \end{vmatrix} [C_1 \exp(-j\beta_1 x) + D_1 \exp(j\beta_1 x)] + \begin{vmatrix} -p_2 i_{21}/\Delta i \\ p_2 i_{11}/\Delta i \end{vmatrix} [C_2 \exp(-j\beta_2 x) + D_2 \exp(j\beta_2 x)] \quad (16)$$

and

$$\begin{vmatrix} I_1(x) \\ I_2(x) \end{vmatrix} = \begin{vmatrix} i_{11}/Z_1 \\ i_{21}/Z_1 \end{vmatrix} [C_1 \exp(-j\beta_1 x) - D_1 \exp(j\beta_1 x)] + \begin{vmatrix} i_{12}/Z_2 \\ i_{22}/Z_2 \end{vmatrix} [C_2 \exp(-j\beta_2 x) - D_2 \exp(j\beta_2 x)]. \quad (17)$$

IV. Self- and Mutual Impedances of the Circuit Model

A characteristic impedance Z_{ij} of a conductor i ($i = 1,2$) for an eigenmode j ($j = 1,2$) propagating in the positive x direction is defined as the ratio of the contribution of eigenmode j to the circuit voltage at conductor i to the contribution of mode j to the circuit current in conductor i. From (16) and (17) it follows that Z_{ij} is given by

$$\begin{aligned} Z_{11} &= (p_1 i_{22} Z_1)/(i_{11}\Delta i) \\ Z_{12} &= -(p_2 i_{21} Z_2)/(i_{12}\Delta i) \\ Z_{21} &= -(p_1 i_{12} Z_1)/(i_{21}\Delta i) \\ Z_{22} &= (p_2 i_{11} Z_2)/(i_{22}\Delta i). \end{aligned} \quad (18)$$

The inconvenience of the above expressions is that i_{ij}, p_j, and Z_j ($i, j = 1,2$) are quantities which do not follow directly from the numerical analysis of the problem; e.g., it is always possible to multiply $\boldsymbol{E}_{t,1}$ in (1) with an arbitrary constant provided $V_{v,1}$ is divided by that same constant. As indicated in [12], the fields in (1) satisfy Maxwell's equations for any value of Z_j in (4). The numerical calculations start from a representation of the unknown currents on the conductors 1 and 2, as will be illustrated in Section VIII for a coupled strip configuration. In solving the eigenvalue problem, the propagation constants β_1 and β_2 are found as eigenvalues. The eigenvectors of the problem yield the eigencurrent distributions on each conductor and this for each eigenmode. The total longitudinal current flowing in the positive x direction on conductor i at $x = 0$ due to eigenmode j is defined as I_{ij} and follows directly from the numerical solution of the problem. From these currents the electric and magnetic fields corresponding to each mode can be found. We denote these fields by $\boldsymbol{E}_{\text{tot},j}$ and $\boldsymbol{H}_{\text{tot},j}$ ($j = 1,2$). The $\boldsymbol{E}$ in (1) is the sum of $\boldsymbol{E}_{\text{tot},1}$ and $\boldsymbol{E}_{\text{tot},2}$, and $\boldsymbol{H}$ in (1) is the sum of $\boldsymbol{H}_{\text{tot},1}$ and $\boldsymbol{H}_{\text{tot},2}$. With these fields it is possible to numerically determine the power associated with eigenmode j propagating in the positive x direction. This power is denoted by P_j ($j = 1,2$) and is given by

$$P_j = 1/2 \iint_S (\boldsymbol{E}_{\text{tot},j} \times \boldsymbol{H}^*_{\text{tot},j}) \cdot d\boldsymbol{S}. \quad (19)$$

From (7), (8), and (11) (but only applied to modes propagating in the positive x direction) it can be found that the following relations hold between i_{ij} and I_{ij}:

$$\frac{I_{1j}}{I_{2j}} = \frac{i_{1j}}{i_{2j}}, \qquad j = 1,2 \quad (20)$$

and between p_j and P_j:

$$2P_j/|I_{ij}|^2 = p_j Z_j/|i_{ij}|^2. \quad (21)$$

Substituting (20) and (21) in (18) leads to

$$\begin{aligned} Z_{11} &= 2(P_1 I_{22})/(I_{11}\Delta I) \\ Z_{12} &= -2(P_2 I_{21})/(I_{12}\Delta I) \\ Z_{21} &= -2(P_1 I_{12})/(I_{21}\Delta I) \\ Z_{22} &= 2(P_2 I_{11})/(I_{22}\Delta I) \end{aligned} \quad (22)$$

with

$$\Delta I = I_{11}I_{22} - I_{12}I_{21}. \quad (23)$$

The above values for the Z_{ij} no longer contain the unknown wave impedances Z_1 and Z_2. To obtain (22) and (23) we again took the real character of I_{ij} and i_{ij} into account. From (22) it is easy to prove that the ratio of the characteristic impedance of conductor 1 to the characteristic impedance of conductor 2 is the same for each eigenmode, i.e.,

$$\frac{Z_{11}}{Z_{21}} = \frac{Z_{12}}{Z_{22}} = -\frac{I_{21}I_{22}}{I_{11}I_{12}}. \tag{24}$$

The above equations also show that this ratio can be expressed in terms of the longitudinal currents. Tripathi [1] shows that (24) results directly from the reciprocity which must be satisfied by the transmission line equations.

Using (22) and (24) the first CTL representation (eqs. (16) and (17)) can be cast in its final form:

$$\begin{vmatrix} V_1(x) \\ V_2(x) \end{vmatrix} = \begin{vmatrix} 1 \\ -I_{12}/I_{22} \end{vmatrix} [A_1 \exp(-j\beta_1 x) + B_1 \exp(j\beta_1 x)] + \begin{vmatrix} 1 \\ -I_{11}/I_{21} \end{vmatrix} \cdot [A_2 \exp(-j\beta_2 x) + B_2 \exp(j\beta_2 x)] \tag{25}$$

and

$$\begin{vmatrix} I_1(x) \\ I_2(x) \end{vmatrix} = \begin{vmatrix} 1/Z_{11} \\ -I_{12}/(I_{22}Z_{21}) \end{vmatrix} \cdot [A_1 \exp(-j\beta_1 x) - B_1 \exp(j\beta_1 x)] + \begin{vmatrix} 1/Z_{12} \\ -I_{11}/(I_{21}Z_{22}) \end{vmatrix} \cdot [A_2 \exp(-j\beta_2 x) - B_2 \exp(j\beta_2 x)]. \tag{26}$$

It is important to remark that the ratios of the I_{ij} currents and the Z_{ij} values used in (25) and (26) are independent of an arbitrary multiplicative factor involved in the choice of the eigencurrents.

V. Distribution of the Power over Each Conductor

We will now investigate what the consequences of the above CTL model are on the distribution of the propagating power over the conductors 1 and 2. Here again we only consider waves propagating in the positive x direction; hence B_1 and B_2 must be set to zero in (25) and (26). These equations then show that the power propagated by each transmission line in the CTL model and denoted by $P_{g,j}$ $(j=1,2)$ is given by

$$\begin{aligned} P_{g,1} = {} & |A_1|^2/(2Z_{11}) + |A_2|^2/(2Z_{12}) \\ & + (A_1 A_2^*/Z_{12}) \exp[-j(\beta_1 - \beta_2)x] \\ & + (A_1^* A_2/Z_{11}) \exp[-j(\beta_2 - \beta_1)x] \end{aligned} \tag{27}$$

and

$$\begin{aligned} P_{g,2} = {} & (|A_1|^2 I_{12}^2)/(2Z_{21}I_{22}^2) + (|A_2|^2 I_{11}^2)/(2Z_{22}I_{21}^2) \\ & + [(A_1 A_2^* I_{12} I_{11})/(2I_{21}I_{22}Z_{22})] \\ & \cdot \exp[-j(\beta_1 - \beta_2)x] \\ & + [(A_1^* A_2 I_{12} I_{11})/(2I_{21}I_{22}Z_{21})] \\ & \cdot \exp[-j(\beta_2 - \beta_1)x]. \end{aligned} \tag{28}$$

In each of the above expressions the first term represents the contribution from eigenmode 1, the second term represents the contribution from eigenmode 2, and the last two terms represent power coupling or cross-power terms. Defining P_{ij} as the part of the power associated with eigenmode j $(j=1,2)$ and transmitted by conductor i $(i=1,2)$ in the absence of the other eigenmode, we have

$$\begin{aligned} P_{11} &= |A_1|^2/(2Z_{11}) \\ P_{21} &= |A_1|^2 I_{12}^2/(2Z_{21}I_{22}^2) \\ P_{12} &= |A_2|^2/(2Z_{12}) \\ P_{22} &= |A_2|^2 I_{11}^2/(2Z_{22}I_{21}^2). \end{aligned} \tag{29}$$

The total power P_j $(j=1,2)$ associated with each mode j in the absence of the other mode is given by

$$\begin{aligned} P_{11} + P_{21} &= P_1 \\ P_{12} + P_{22} &= P_2 \end{aligned} \tag{30}$$

while the total power P propagated in the presence of both eigenmodes is

$$P = P_{11} + P_{12} + P_{21} + P_{22}. \tag{31}$$

The fact that the power coupling terms in (27) and (28) disappear from the total power budget is a consequence of the power orthogonality of the modes. The two relations given by (30) allow us to eliminate the coefficients A_1 and A_2 from (29). This leads to

$$\begin{aligned} P_{11} &= \frac{P_1 I_{11} I_{22}}{\Delta I} \\ P_{21} &= -\frac{P_1 I_{21} I_{12}}{\Delta I} \\ P_{12} &= -\frac{P_2 I_{21} I_{12}}{\Delta I} \\ P_{22} &= \frac{P_2 I_{11} I_{22}}{\Delta I}. \end{aligned} \tag{32}$$

The expressions in (32) finally lead to the identity

$$P_{11}/P_{21} = P_{12}/P_{22}. \tag{33}$$

This relation expresses that, e.g., if 30 percent of the power of eigenmode 1 is propagated by line 1 and 70 percent by line 2, it automatically follows that 70 percent of the power of eigenmode 2 will be propagated by line 1 and 30 percent by line 2. This reasoning holds only if each eigenmode is considered separately. If both eigenmodes are present at the same time, part of the power oscillates between line 1 and 2 as a function of the longitudinal distance along the waveguide structure. The typical length involved in this process is the coupling length between the modes as the power coupling terms in (28) and (29) contain a phase factor depending upon the difference of the propagation constants of modes 1 and 2.

In the past Jansen [4] has proposed the following expression for the partial power values P_{ij}:

$$P_{ij} = (1/2) \iint_S (\boldsymbol{E}_j \times \boldsymbol{H}_{i,j}^*) \cdot d\boldsymbol{S}. \tag{34}$$

In (34) $\boldsymbol{E}_j$ represents the total electric field associated with eigenmode j, where, as above, only a wave propagating in the positive x direction is considered. The magnetic field $\boldsymbol{H}_{t,j}$ is defined as the magnetic field due to the current on conductor i while the current on the other conductor is zero, again for eigenmode j. It can be shown that the values of P_{ij} obtained by applying (34) are exact in the low-frequency or quasi-TEM limit. The expression (34) seems to be inspired by a closely related one, i.e.,

$$P_{ij} = (1/2)\iint_S (\boldsymbol{E}_{i,j} \times \boldsymbol{H}_{i,j}^*) \cdot d\boldsymbol{S}. \tag{35}$$

In (35) the definition of $\boldsymbol{H}_{i,j}$ is the same as in (34), and $\boldsymbol{E}_{i,j}$ is defined as the electric field for eigenmode j due to the voltage on conductor i, while the voltage on the other conductor is zero. It is obvious that the use of voltages restricts the validity of the definition to the low-frequency or quasi-TEM limit. In that case the longitudinal field components are negligibly small with respect to the transversal ones, and both the electric and magnetic field can be derived from a potential and are perpendicular to each other at each point of the cross section. In that particular case (35) is exact. Since a typical full-wave analysis using the spectral-domain method yields the total electric field and the partial magnetic fields, an obvious extension of (35) is given by (34). In the quasi-TEM limit, however, (34) and (35) yield the same result.

We will now show which fields are involved in the expressions for P_{ij} (32) proposed in this paper. To this end the field currents and voltages in (1) are replaced by circuit quantities using (7) and (8). This leads to

$$\begin{aligned} \boldsymbol{E}(x,y,z) = {} & [(v_{22}\boldsymbol{E}_{t,1} - v_{21}\boldsymbol{E}_{t,2})/\Delta v]\, V_1(x) \\ & + [(-v_{12}\boldsymbol{E}_{t,1} + v_{11}\boldsymbol{E}_{t,2})/\Delta v]\, V_2(x) \\ & + [R_0(i_{22}\boldsymbol{E}_{l,1} - i_{21}\boldsymbol{E}_{l,2})/\Delta i]\, I_1(x) \\ & + [R_0(-i_{12}\boldsymbol{E}_{l,1} + i_{11}\boldsymbol{E}_{l,2})/\Delta i]\, I_2(x) \end{aligned} \tag{36}$$

and

$$\begin{aligned} \boldsymbol{H}(x,y,z) = {} & [(i_{22}\boldsymbol{H}_{t,1} - i_{21}\boldsymbol{H}_{t,2})/\Delta i]\, I_1(x) \\ & + [(-i_{12}\boldsymbol{H}_{t,1} + i_{11}\boldsymbol{H}_{t,2})/\Delta i]\, I_2(x) \\ & + [1/R_0(v_{22}\boldsymbol{H}_{l,1} - v_{21}\boldsymbol{H}_{l,2})/\Delta v]\, V_1(x) \\ & + [1/R_0(-v_{12}\boldsymbol{H}_{l,1} + v_{11}\boldsymbol{H}_{l,2})/\Delta v]\, V_2(x). \end{aligned} \tag{37}$$

The quantity Δv is given by

$$\Delta v = v_{11}v_{22} - v_{12}v_{21} = p_1 p_2/\Delta i. \tag{38}$$

The last equality in (38) can be derived from (14). Starting from (25) and (26) and using (36) and (37) and the definition of P_{ij}, it is found that P_{ij} is given by

$$P_{ij} = (1/2)\iint_S (\boldsymbol{E}_{a,i,j} \times \boldsymbol{H}_{a,i,j}^*) \cdot d\boldsymbol{S}. \tag{39}$$

The index a indicates that the fields in (39) represent some kind of average. For conductor 1 they are given by

$$\begin{aligned} \boldsymbol{E}_{a,1,j} &= Z_{1j}(v_{22}\boldsymbol{E}_{t,1} - v_{21}\boldsymbol{E}_{t,2})I_{1j}/\Delta v \\ \boldsymbol{H}_{a,1,j} &= (i_{22}\boldsymbol{H}_{t,1} - i_{21}\boldsymbol{H}_{t,2})I_{1j}/\Delta i. \end{aligned} \tag{40}$$

For conductor 2 we have

$$\begin{aligned} \boldsymbol{E}_{a,2,j} &= Z_{2j}(-v_{12}\boldsymbol{E}_{t,1} + v_{11}\boldsymbol{E}_{t,2})I_{2j}/\Delta v \\ \boldsymbol{H}_{a,2,j} &= (-i_{12}\boldsymbol{H}_{t,1} + i_{11}\boldsymbol{H}_{t,2})I_{2j}/\Delta i. \end{aligned} \tag{41}$$

In order to be able to compare (39) with either (34) or (35), it is necessary to recast all quantities in (39) in terms of quantities which follow directly from the numerical solution of a particular problem. To do so we again use I_{ij}, Z_{ij}, $\boldsymbol{E}_{\text{tot},j}$, and $\boldsymbol{H}_{\text{tot},j}$, which have already been defined, and also $V_{ij} = Z_{ij}I_{ij}$. It can easily be shown that (40) and (41) can be rewritten as

$$\begin{aligned} \boldsymbol{E}_{a,1,j} &= Z_{1j}(V_{22}\boldsymbol{E}_{\text{tot},1} - V_{21}\boldsymbol{E}_{\text{tot},2})I_{1j}/\Delta V \\ \boldsymbol{H}_{a,1,j} &= (I_{22}\boldsymbol{H}_{\text{tot},1} - I_{21}\boldsymbol{H}_{\text{tot},2})I_{1j}/\Delta I \\ \boldsymbol{E}_{a,2,j} &= Z_{2j}(-V_{12}\boldsymbol{E}_{\text{tot},1} + V_{11}\boldsymbol{E}_{\text{tot},2})I_{2j}/\Delta V \\ \boldsymbol{H}_{a,2,j} &= (-I_{12}\boldsymbol{H}_{\text{tot},1} + I_{11}\boldsymbol{H}_{\text{tot},2})I_{2j}/\Delta I. \end{aligned} \tag{42}$$

The quantity ΔI is defined in (23) and ΔV is defined as in (38). Compared with (34) and (35), the actual fields used here to calculate P_{ij} are well-chosen combinations of the fields of both modes, whereas in (34) and (35) only the (partial) fields due to a certain mode are used. It is again possible to prove that (42) is equivalent to (34) and (35) in the quasi-TEM limit. The proof is beyond the scope of this paper. The equalities in (24), which are a direct consequence of the reciprocity, are automatically satisfied in our model. These equalities will no longer be satisfied if one starts from (34) for calculating the partial powers P_{ij}.

VI. Extension of the Circuit Representation to Lossy Structures

The main difference between the analysis for lossless waveguides and lossy ones resides in the fact that the power orthogonality (eq. (5)) is no longer satisfied and that the cross-powers p_{ij} defined by

$$p_{ij} = \iint_S (\boldsymbol{E}_{t,i} \times \boldsymbol{H}_{t,j}^*) \cdot d\boldsymbol{S}, \qquad i,j=1,2 \;\; i \neq j \tag{43}$$

have to be introduced. Similar reasoning leads to the following relations between v_{ij} and i_{ij}:

$$\begin{aligned} v_{11}i_{11}^* + v_{21}i_{21}^* &= p_1 \\ v_{12}i_{12}^* + v_{22}i_{22}^* &= p_2 \\ v_{11}i_{12}^* + v_{21}i_{22}^* &= p_{12} \\ v_{12}i_{11}^* + v_{22}i_{21}^* &= p_{21}. \end{aligned} \tag{44}$$

These expressions replace the ones given by (13). As the transversal currents can no longer be chosen to be real, we have introduced the necessary complex conjugates. Solving for v_{ij} gives

$$\begin{aligned} v_{11} &= (p_1 i_{22}^* - p_{12}i_{21}^*)/\Delta i^* \\ v_{21} &= (-p_1 i_{12}^* + p_{12}i_{11}^*)/\Delta i^* \\ v_{12} &= (-p_2 i_{21}^* + p_{21}i_{22}^*)/\Delta i^* \\ v_{22} &= (p_2 i_{11}^* - p_{21}i_{12}^*)/\Delta i^*. \end{aligned} \tag{45}$$

Using

$$Z_{ij} = Z_j v_{ij}/i_{ij} \tag{46}$$

the values of Z_{ij} now turn out to be

$$Z_{11} = (p_1 i_{22}^* Z_1 - p_{12} i_{21}^* Z_1)/(i_{11}\Delta i^*)$$
$$Z_{12} = (-p_2 i_{21}^* Z_2 + p_{21} i_{22}^* Z_2)/(i_{12}\Delta i^*)$$
$$Z_{21} = (-p_1 i_{12}^* Z_1 + p_{12} i_{11}^* Z_1)/(i_{21}\Delta i^*)$$
$$Z_{22} = (p_2 i_{11}^* Z_2 - p_{21} i_{12}^* Z_2)/(i_{22}\Delta i^*). \tag{47}$$

The next step consists again in expressing all relevant quantities in terms of values which directly follow from the numerical solution of the problem. The relations which can be used for that purpose are (21) and

$$2P_{ij}/(I_{kj}^* I_{ki}) = p_{ij} Z_i/(i_{kj}^* i_{ki}), \qquad i, j, k = 1,2 \quad \text{and} \quad i \neq j \tag{48}$$

where the I_{ij} and P_j are defined as before and P_{ij} is given by

$$P_{ij} = 1/2 \iint_S (\boldsymbol{E}_{\text{tot},i} \times \boldsymbol{H}_{\text{tot},j}^*), \qquad j \neq i. \tag{49}$$

Using (21) and (48), (47) can be rewritten as

$$Z_{11} = 2(P_1 I_{22}^* - P_{12} I_{21}^*)/(I_{11}\Delta I^*)$$
$$Z_{12} = 2(-P_2 I_{21}^* + P_{21} I_{22}^*)/(I_{12}\Delta I^*)$$
$$Z_{21} = 2(-P_1 I_{12}^* + P_{12} I_{11}^*)/(I_{21}\Delta I^*)$$
$$Z_{22} = 2(P_2 I_{11}^* - P_{21} I_{12}^*)/(I_{22}\Delta I^*). \tag{50}$$

The CTL representation (25) and (26) for lossless lines finally becomes

$$\begin{vmatrix} V_1(x) \\ V_2(x) \end{vmatrix} = \begin{vmatrix} 1 \\ -I_{12}^* C_1/I_{22}^* \end{vmatrix} [A_1 \exp(-j\beta_1 x) + B_1 \exp(j\beta_1 x)] + \begin{vmatrix} 1 \\ -I_{11}^* C_2/I_{21}^* \end{vmatrix} \cdot [A_2 \exp(-j\beta_2 x) + B_2 \exp(j\beta_2 x)] \tag{51}$$

and

$$\begin{vmatrix} I_1(x) \\ I_2(x) \end{vmatrix} = \begin{vmatrix} 1/Z_{11} \\ -I_{12}^* C_1/(I_{22}^* Z_{21}) \end{vmatrix} \cdot [A_1 \exp(-j\beta_1 x) - B_1 \exp(j\beta_1 bx)] + \begin{vmatrix} 1/Z_{12} \\ -I_{11}^* C_2/(I_{21}^* Z_{22}) \end{vmatrix} \cdot [A_2 \exp(-j\beta_2 x) - B_2 \exp(j\beta_2 x)]. \tag{52}$$

The coefficients C_1 and C_2 are given by

$$C_1 = [1 - (P_{12} I_{11}^*)/(P_1 I_{12}^*)]/[1 - (P_{12} I_{21}^*)/(P_1 I_{22}^*)]$$
$$C_2 = [1 - (P_{21} I_{12}^*)/(P_2 I_{11}^*)]/[1 - (P_{21} I_{22}^*)/(P_2 I_{21}^*)]. \tag{53}$$

C_1 and C_2 introduce the correction of the CTL model in the presence of losses while β_1 and β_2 are now complex propagation constants.

VII. Extension of the Circuit Model to N Coupled Lines

In this section we briefly deal with the extension of the above CTL model to the case of the N fundamental modes of a multiconductor waveguide. The equality between the power propagated by the modes and the power propagated in the CTL model leads to

$$\sum_{i=1}^{i=N} v_{ij} i_{ij}^* = p_j, \qquad j = 1, \cdots, N$$
$$\sum_{i=1}^{i=N} v_{ij} i_{ik}^* = p_{jk}, \qquad k \neq j \quad \text{and } j, k = 1, \cdots, N \tag{54}$$

where

$$p_i = \iint_S (\boldsymbol{E}_i \times \boldsymbol{H}_i^*) \cdot d\boldsymbol{S}$$
$$p_{jk} = \iint_S (\boldsymbol{E}_j \times \boldsymbol{H}_k^*) \cdot d\boldsymbol{S}. \tag{55}$$

As for two coupled lines, we introduce the current I_{ij}, which is the total longitudinal current flowing along conductor i at $x = 0$ due to eigenmode j. We also introduce P_j, the power propagated by eigenmode j at $x = 0$, and P_{jk}, the cross-power due to the electric field of mode j and the magnetic field of mode k, both taken at $x = 0$. It can be shown that (54) is equivalent to

$$\sum_{i=1}^{i=N} V_{ij} I_{ij}^* = P_j, \qquad j = 1, \cdots, N$$
$$\sum_{i=1}^{i=N} V_{ij} I_{ik}^* = P_{jk}, \qquad k \neq j \text{ and } j, k = 1, \cdots, N \tag{56}$$

where the voltages V_{ij} do not have a direct physical meaning. Solving (56), the mathematical quantities V_{ij} can be determined, leading to the Z_{ij} values using $Z_{ij} = V_{ij}/I_{ij}$. The final CTL model then becomes

$$V_i(x) = \sum_{j=1}^{j=N} [(Z_{ij} I_{ij})/(Z_{1j} I_{1j})] \cdot [A_j \exp(-j\beta_j x) + B_j \exp(j\beta_j x)]$$
$$I_i(x) = \sum_{j=1}^{j=N} [I_{ij}/(I_{1j} Z_{1j})] \cdot [A_j \exp(-j\beta_j x) + B_j \exp(j\beta_j x)]. \tag{57}$$

VIII. Numerical Example

In this section we consider the typical coupled microstrip configuration shown in Fig. 2. The value of h is 1 mm. This example is also discussed in [7]. Using the space-domain Green's function approach discussed in [16] and [14], the eigenmode problem is solved. Numerical calculation of the currents I_{ij}, $j = 1,2$, and of the powers P_1 and P_2, discussed in Section IV, leads to the impedances Z_{ij} (22). The solid lines in Fig. 3 show these impedances. The first index i refers to the conductor, while the second

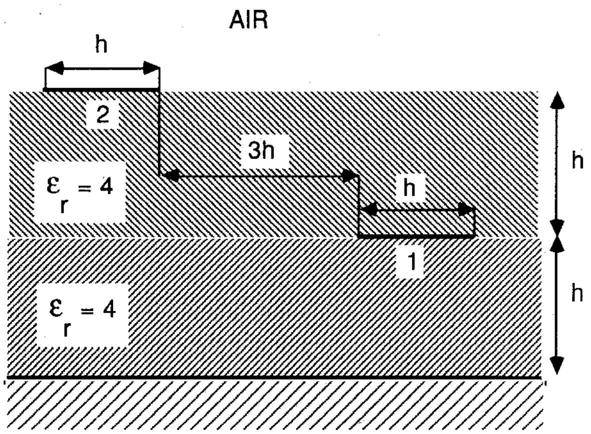

Fig. 2. A typical coupled microstrip configuration ($h = 1$ mm).

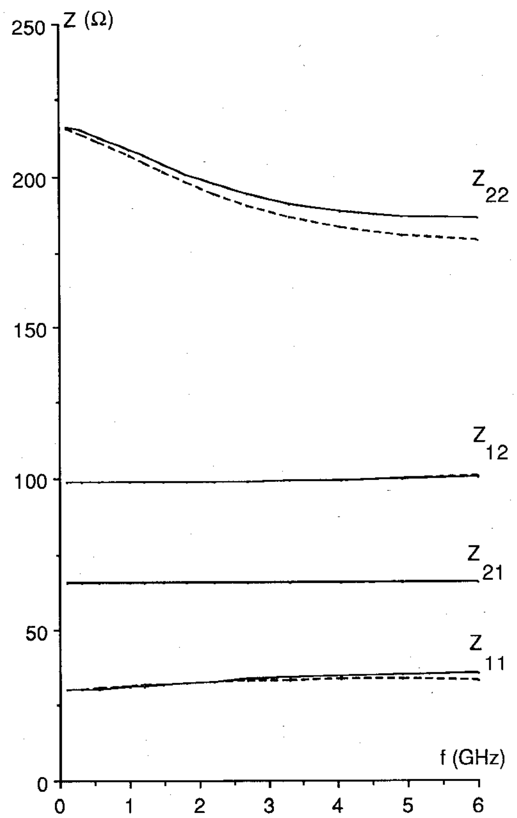

Fig. 3. Characteristic impedances as a function of the frequency for the configuration of Fig. 2. Solid lines: our results. Dashed lines: results from Jansen's definition (eq. (34)).

index j refers to the mode. Mode 1 corresponds to the quasi-odd mode while mode 2 corresponds to the quasi-even mode. We then calculated the characteristic impedances using the power distribution proposed by Jansen (eq. (34)). The differences between Jansen's approach and ours increase as a function of frequency. Special care was taken to perform all calculations in double precision arithmetic, avoiding the possibility of different results being generated by numerical imprecision. For Z_{12} and Z_{21}, however, the difference remains very small. As a check on relation (24), which is a consequence of the reciprocity, we calculated the following variable:

$$\gamma = |1-(Z_{11}Z_{22})/(Z_{12}Z_{21})| \tag{58}$$

which must be zero. Fig. 4 shows γ as a function of frequency using Jansen's definition (eq. (34)). This figure clearly shows that the approach based on (34) yields a correct circuit model only in the quasi-TEM limit.

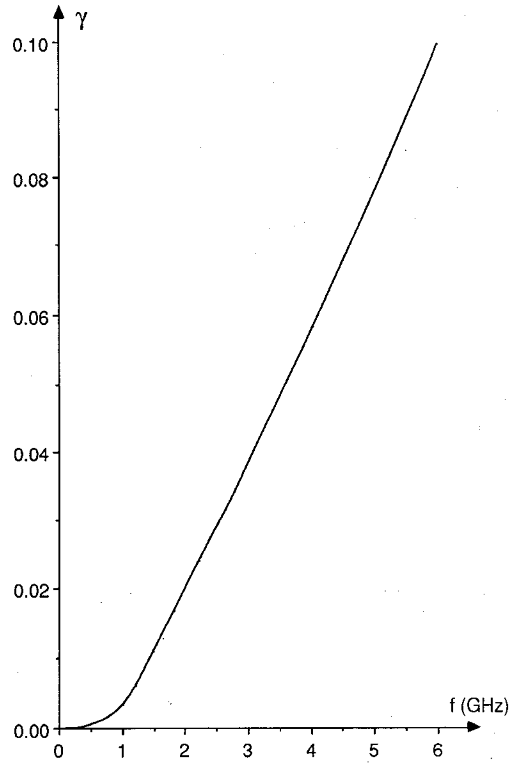

Fig. 4. γ as a function of the frequency using the power definition of Jansen (eq. (34)) applied to the configuration of Fig. 2.

IX. Conclusion

In this paper a new high-frequency model for N coupled lossless and lossy waveguides was presented. A coupled transmission line model of such structures, for both the nondispersive and the dispersive case, was found based on a power–current formulation of the impedances. To actually be able to determine these impedances, the full-wave eigenmode problem for the waveguides under consideration and for the N lowest eigenmodes must be solved. It does not suffice to determine the propagation constants belonging to each mode. One also needs the power distribution over the total cross section of the waveguide belonging to each eigenmode. However, no supposition about the power distribution over each separate conductor is needed.

We have also shown, theoretically and by a typical example, that our new approach coincides with previously adopted approaches in the low-frequency limit but leads to different circuit parameters for coupled waveguide structures at high frequencies. This will in turn influence the values obtained in, e.g., crosstalk calculations, especially if very high speed digital signals with a considerable high-frequency content are involved.

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